Contents

[Adding noise 2](#_Toc110325574)

[Solving in SINDy-PI 3](#_Toc110325575)

[Changing the library for ensemble-sindy 5](#_Toc110325576)

# Adding noise

|  |  |  |  |
| --- | --- | --- | --- |
| **Objective** | Add noise to the ode, then solve as normal | | |
| **Test** | **Result** | **Output** | **Comment** |
| Solve spring mass damper & sin system w/out noise | This is the baseline to compare to noise |  |  |
| Solving the same system and adding noise (to solution)   * Mu=0, sigma=100 | The same response, just shifted up |  | The noise is just a constant term added to the solution so it has just shifted everything up   * How to incorporate noise ? |
| Solving the same system and adding noise to the odes (the 2nd one in the function)   * Mu=0, sigma=100 | More smooth, negative disp and vel |  |  |

|  |  |  |  |
| --- | --- | --- | --- |
| **Objective** | Add noise to the solutions individually | | |
| Adding noise to solutions  (to disp, vel, & accel)   * Random noise | Follows the same rough shape, just noisier (as expected) |  | This looks more like what I was expecting |
| Rmse noise (1%) | Less noisy than using random noise |  |  |

# Solving in SINDy-PI

|  |  |  |  |
| --- | --- | --- | --- |
| **Objective** | Adding noise and solving with SINDy-PI | | |
| Added noise using root mean squared error (from pysindy ex 13 emsembling) | Did not recover dynamics | x0\_dot = 9.248 x0 + -1.620 x1 + 0.286 x0x1 + -13.736 x0x0 + 0.868 x1x1 + -0.189 x0x1x1 + 7.169 x0x0x0 + -0.045 x1x1x1 + 0.012 x0x1x1x1 + -1.708 x0x0x0x0 + 0.095 x1x1x1x1 + 0.190 x0x0x0x0x0 + 0.029 x1x1x1x1x1 + -0.008 x0x0x0x0x0x0 + -0.023 x1x1x1x1x1x1 + -0.314 x1\_dot + 0.388 x0x0\_dot + 0.299 x1x0\_dot + -0.055 x0x1x0\_dot + 0.323 x0x0x0\_dot + 0.206 x1x1x0\_dot + -0.034 x0x1x1x0\_dot + -0.270 x0x0x0x0\_dot + -0.092 x1x1x1x0\_dot + 0.014 x0x1x1x1x0\_dot + 0.075 x0x0x0x0x0\_dot + -0.017 x1x1x1x1x0\_dot + -0.009 x0x0x0x0x0x0\_dot + 0.003 x1x1x1x1x1x0\_dot + 0.001 x1x1x1x1x1x1x0\_dot + 0.093 x0x1\_dot + 0.174 x1x1\_dot + -0.031 x0x1x1\_dot + 0.095 x0x0x1\_dot + 0.034 x1x1x1\_dot + -0.006 x0x1x1x1\_dot + -0.066 x0x0x0x1\_dot + -0.045 x1x1x1x1\_dot + 0.005 x0x1x1x1x1\_dot + 0.017 x0x0x0x0x1\_dot + -0.001 x1x1x1x1x1\_dot + -0.002 x0x0x0x0x0x1\_dot + 0.003 x1x1x1x1x1x1\_dot  x1\_dot = 8.696 x0 + 0.697 x1 + -0.168 x0x1 + -12.411 x0x0 + -0.840 x1x1 + 0.033 x0x1x1 + 6.544 x0x0x0 + -0.235 x1x1x1 + 0.037 x0x1x1x1 + -1.597 x0x0x0x0 + 0.236 x1x1x1x1 + 0.182 x0x0x0x0x0 + 0.020 x1x1x1x1x1 + -0.008 x0x0x0x0x0x0 + -0.018 x1x1x1x1x1x1 + -0.294 x0\_dot + 0.042 x0x0\_dot + 0.150 x1x0\_dot + -0.027 x0x1x0\_dot + 0.196 x0x0x0\_dot + 0.022 x1x1x0\_dot + -0.003 x0x1x1x0\_dot + -0.129 x0x0x0x0\_dot + -0.033 x1x1x1x0\_dot + 0.004 x0x1x1x1x0\_dot + 0.033 x0x0x0x0x0\_dot + -0.003 x1x1x1x1x0\_dot + -0.004 x0x0x0x0x0x0\_dot + 0.002 x1x1x1x1x1x0\_dot + 0.457 x0x1\_dot + 0.283 x1x1\_dot + -0.054 x0x1x1\_dot + 0.182 x0x0x1\_dot + 0.211 x1x1x1\_dot + -0.035 x0x1x1x1\_dot + -0.184 x0x0x0x1\_dot + -0.080 x1x1x1x1\_dot + 0.014 x0x1x1x1x1\_dot + 0.052 x0x0x0x0x1\_dot + -0.015 x1x1x1x1x1\_dot + -0.006 x0x0x0x0x0x1\_dot + 0.001 x1x1x1x1x1x1\_dot + 0.001 x1x1x1x1x1x1x1\_dot | Figures = model over actual dynamics, actual dynamics, model  Equations have lots of terms – not sparse |
| Reduce rmse (from 1% to 0.1%) | Did not recover dynamics (modelled the noise) |  | Can see more of the shape with this amount of noise – looks a bit closer to noise less dynamics |
| Using ensemble-SINDy |  | Ensemble without replacement (V2)  (x0)' = 1.167 1 + -0.899 x0 + -0.710 x1 + 0.133 x0^2 + 0.356 x0 x1 + 0.646 x1^2  (x1)' = 3.224 1 + -0.645 x0 + -1.286 x1 | Much sparser equations |
| Plotting ensemble model | It did determine the dynamics (? – still a bit noisy) | Ensemble without replacement (V2)  (x0)' = -0.541 1 + 0.835 x0 + 1.955 x1 + -0.144 x0^2 + -0.254 x0 x1 + -0.480 x1^2  (x1)' = 3.854 1 + -0.767 x0 + -2.092 x1 + 0.105 x0 x1 + 0.225 x1^2 | Unsure what library has been used and therefore whether it contains derivative terms  *The default library is polynomial (which itself has the default of degree=2)* |
| Repeating with spring damper & sin term system | Recovered the dynamics  (again, still a little noisy) | Ensemble without replacement (V2)  (x0)' = 0.998 x1  (x1)' = 2.544 1 + -0.508 x0 + 0.510 x1 + -0.154 x0 x1 + -0.997 x1^2 |  |
| Plot ensemble-sindy model on top of actual dynamics (noiseless) | Shows how close to actual solution it is |  | Need to look at library next – change to custom w/ polynomials, sin and derivatives |

# Changing the library for ensemble-sindy

(01/08 from looking at the case western data I’ve realised I wasn’t actually changing the library when I thought I was)

|  |  |  |  |
| --- | --- | --- | --- |
| **Method** | Specifying the library | | |
| **Test** | **Output** | | **Comment** |
| Ensemble-SINDy  Adding 1% rmse noise  Polynomial library | Noisy  Library:  ['1', 'x0', 'x1', 'x0^2', 'x0 x1', 'x1^2']    No ensembling  (x0)' = 0.936 1 + -0.406 x0 + -0.351 x1 + 0.043 x0^2 + 0.260 x0 x1 + 0.437 x1^2  (x1)' = 2.603 1 + -0.262 x0 + 0.847 x1 + -0.050 x0^2 + -0.208 x0 x1 + -1.344 x1^2  Ensemble with replacement (V1)  (x0)' = 0.936 1 + -0.406 x0 + -0.351 x1 + 0.043 x0^2 + 0.260 x0 x1 + 0.437 x1^2  (x1)' = 2.603 1 + -0.262 x0 + 0.847 x1 + -0.050 x0^2 + -0.208 x0 x1 + -1.344 x1^2  Ensemble without replacement (V2)  (x0)' = 0.936 1 + -0.406 x0 + -0.351 x1 + 0.043 x0^2 + 0.260 x0 x1 + 0.437 x1^2  (x1)' = 2.603 1 + -0.262 x0 + 0.847 x1 + -0.050 x0^2 + -0.208 x0 x1 + -1.344 x1^2  Library Ensemble (V3)  (x0)' = 0.936 1 + -0.406 x0 + -0.351 x1 + 0.043 x0^2 + 0.260 x0 x1 + 0.437 x1^2  (x1)' = 3.015 1 + -0.468 x0 + -0.026 x0^2 + -0.046 x0 x1 + -1.016 x1^2  Combination Ensemble (V4)  (x0)' = 0.936 1 + -0.406 x0 + -0.351 x1 + 0.043 x0^2 + 0.260 x0 x1 + 0.437 x1^2  (x1)' = 3.015 1 + -0.468 x0 + -0.026 x0^2 + -0.046 x0 x1 + -1.016 x1^2 | | Note that the plot looks the same for all 5 models |
| Changing the spring-mass-sin in original SINDy to ensemble SINDy  w/ polynomial (deg =2) library | Original |  |  |
| Ensemble  (not incl. x\_dot in SINDy model) | Model (w/ Ensemble)  (x0)' = 5.298 1 + 1.801 x0 + -7.382 x1 + -0.954 x0^2 + -1.158 x0 x1 + -2.455 x1^2 + 0.080 x0^3 + 0.372 x0^2 x1 + 0.411 x0 x1^2 + 0.648 x1^3  (x1)' = 7.640 1 + -16.554 x0 + -10.580 x1 + 6.356 x0^2 + 3.508 x0 x1 + -3.813 x1^2 + -0.602 x0^3 + -0.195 x0^2 x1 + 0.369 x0 x1^2 + -0.164 x1^3 |
|  | Ensemble + incl. x\_dot | Model (w/ Ensemble)  (x0)' = 1.000 x1  (x1)' = 3.333 1 + -0.667 x0 + -1.333 x1 |  |
| Ensemble SINDy  Polynomial library  Changed threshold from 1e-3 to .05 | Noisy  Library:  ['1', 'x0', 'x1', 'x0^2', 'x0 x1', 'x1^2']    No ensembling  (x0)' = 0.280 1 + -0.056 x0 + 0.874 x1  (x1)' = 3.177 1 + -0.631 x0 + -0.113 x1 + -0.968 x1^2    Ensemble with replacement (V1)  (x0)' = 0.439 1 + -0.087 x0 + 0.712 x1 + 0.037 x0 x1  (x1)' = 2.516 1 + -0.246 x0 + 0.913 x1 + -0.050 x0^2 + -0.204 x0 x1 + -1.351 x1^2    Ensemble without replacement (V2)  (x0)' = 0.456 1 + -0.091 x0 + 0.628 x1 + 0.052 x0 x1 + 0.038 x1^2  (x1)' = 3.159 1 + -0.627 x0 + -0.072 x1 + -0.008 x0 x1 + -0.982 x1^2    Library Ensemble (V3)  (x0)' = 0.280 1 + -0.056 x0 + 0.874 x1  (x1)' = 3.177 1 + -0.631 x0 + -0.113 x1 + -0.968 x1^2    Combination Ensemble (V4)  (x0)' = 0.004 1 + 1.132 x1 + -0.045 x0 x1 + -0.047 x1^2  (x1)' = 2.516 1 + -0.246 x0 + 0.913 x1 + -0.050 x0^2 + -0.204 x0 x1 + -1.351 x1^2 | | Getting different models for the different ensembling methods  Had to change threshold from 0.05 to 0.005 for V4 to work |

01/08: I wasn’t consistent with what I called the library – originally “library” but then renames in “lib” to fit & transform it, but then called “library” in the sindy model so that’s why it didn’t work (I think)

|  |  |  |
| --- | --- | --- |
| Test | Output | Comment |
| Ensemble sindy  Custom library  Library:  [1, 1, 'x0', 'x1', 'x0^2', 'x1^2'] | No ensembling  (x0)' = 0.989 x1  (x1)' = 1.693 1 + 1.693 1 + -0.672 x0 + -0.118 x1 + -1.036 x1^2    Ensemble with replacement (V1)  (x0)' = 0.095 1 + 0.095 1 + -0.037 x0 + 0.952 x1 + -0.023 x1^2  (x1)' = 1.693 1 + 1.693 1 + -0.672 x0 + -0.118 x1 + -1.036 x1^2    Ensemble without replacement (V2)  (x0)' = 0.989 x1  (x1)' = 1.624 1 + 1.624 1 + -0.490 x0 + -0.128 x1 + -0.031 x0^2 + -1.065 x1^2    Library Ensemble (V3)  (x0)' = 0.989 x1  (x1)' = 1.693 1 + 1.693 1 + -0.672 x0 + -0.118 x1 + -1.036 x1^2    Combination Ensemble (V4)  (x0)' = 0.989 x1  (x1)' = 1.693 1 + 1.693 1 + -0.672 x0 + -0.118 x1 + -1.036 x1^2 | Able to use a custom library (w/ no sin term) |
| Add higher polynomials to see if that is a better expansion of the sin term  Library:  [1, 1, 'x0', 'x1', 'x0^2', 'x1^2', 'x0^3', 'x1^3', 'x0^4', 'x1^4', 'x0^5', 'x1^5'] | Ensemble with replacement (V1)  (x0)' = 0.090 1 + 0.090 1 + 0.052 x0 + 0.720 x1 + -0.038 x0^2 + -0.660 x1^2 + 0.004 x0^3 + 3.688 x1^3 + -4.167 x1^4 + 1.361 x1^5  (x1)' = 1.646 1 + 1.646 1 + -1.660 x0 + 0.209 x1 + 0.451 x0^2 + -0.810 x1^2 + -0.050 x0^3 + -4.653 x1^3 + 5.641 x1^4 + -1.713 x1^5 | Adding the higher order polynomials better approximates the sin (assuming equivalent to an expansion) |
|  |  |  |
|  |  |  |

Thinking it might only be SINDy-PI that can use sin(t) terms